

## Esercizi di riepilogo sulle equazioni

Risolvere le seguenti equazioni.

**299**  $2 \sin^2 \frac{x}{2} + \cos x = 3 \tan^2 x$

$$x = \pm \frac{\pi}{6} + k\pi$$

**300**  $\sin x \tan \frac{x}{2} - \cos x = 0$

$$x = \pm \frac{\pi}{3} + k \cdot 2\pi$$

**301**  $\frac{1 - \cos 2x}{2 \sin x} + \cos x = 1$

$$x = \frac{\pi}{2} + 2k\pi$$

**302**  $2 \sin^2 2x - 6 \sin x \cos x + 1 = 0$

$$x = \frac{\pi}{4} + k\pi; x = \frac{\pi}{12} + k\pi; x = \frac{5}{12}\pi + k\pi$$

**303**  $\tan^2 x + 2 \cos x - 2 = 0$

$$x = 2k\pi; x = \pm \frac{2}{3}\pi + 2k\pi$$

**304**  $\cot^2 x - 8 \sin x + 1 = 0$

$$x = \frac{\pi}{6} + 2k\pi; x = \frac{5}{6}\pi + 2k\pi$$

$$305 \cos\left(3x - \frac{\pi}{3}\right) + \sin\left(\frac{5}{6}\pi - 3x\right) + 1 = 0$$

$$x = -\frac{\pi}{9} + k \cdot \frac{2\pi}{3}; \quad x = \frac{\pi}{3} + k \cdot \frac{2\pi}{3}$$

$$306 \sin\left(x + \frac{2}{3}\pi\right) + \cos\left(\frac{\pi}{6} - x\right) - \sin x - 1 = 0$$

$$x = \frac{\pi}{6} + k \cdot 2\pi; \quad x = -\frac{\pi}{2} + k \cdot 2\pi$$

$$307 \sin^2 x + (1 - \sqrt{3}) \sin x \cdot \cos x - \sqrt{3} \cos^2 x = 0$$

$$x = -\frac{\pi}{4} + k\pi; \quad x = \frac{\pi}{3} + k\pi$$

$$308 \sin x (\tan x - 1) = \sqrt{3} (\sin x - \cos x)$$

$$x = \frac{\pi}{4} + k\pi; \quad x = \frac{\pi}{3} + k\pi$$

$$309 2 \cos^2 2x - \sin 4x = 0$$

$$x = \frac{\pi}{4} + k \frac{\pi}{2}; \quad x = \frac{\pi}{8} + k \frac{\pi}{2}$$

$$310 3 \operatorname{ctg}^2 x + 1 = \frac{3 \cos x}{\cos^2 x - 1}$$

$$x = \pm \frac{2}{3}\pi + 2k\pi$$

$$311 \frac{\sin 2x + 1}{\sin x + \cos x} = \frac{\sqrt{3} - 1}{2}$$

$$x = -\frac{\pi}{6} + 2k\pi; \quad x = \frac{2}{3}\pi + 2k\pi$$

$$312 \frac{3}{\cos x + 1} - \frac{3}{\cos x - 1} = 8$$

$$x = \pm \frac{\pi}{3} + k\pi$$

$$313 \frac{1 - \cos 2x}{\sin x} = \frac{\sin 2x}{1 + \cos 2x}$$

$$x = \pm \frac{\pi}{3} + 2k\pi$$

$$314 \frac{1 - \cos 2x}{\sin x} - \frac{1 + \cos 2x}{\cos x} = 0$$

$$x = \frac{\pi}{4} + k\pi$$

$$315 \sin^2 2x - 4 \sin 2x \cos 2x - 3 \cos^2 2x - 1 = 0$$

$$x = \frac{\pi}{4} + k \frac{\pi}{2}; \quad x = -\frac{\pi}{8} + k \frac{\pi}{2}$$

$$316 3 \sin^2 2x - \sqrt{3} \sin 2x \cos 2x + 2 \cos^2 2x - 3 = 0$$

$$x = \frac{5}{12}\pi + k \frac{\pi}{2}; \quad x = \frac{\pi}{4} + k \frac{\pi}{2}$$

$$317 \operatorname{tg}\left(\frac{\pi}{3} + x\right) = \operatorname{tg}\left(\frac{\pi}{3} - x\right) + 4$$

$$x = \frac{3}{4}\pi + k\pi; \quad x = \arctg \frac{1}{3}$$

$$318 \sin\left(\frac{2}{3}\pi - 2x\right) + \cos 2x = \frac{1}{2}$$

$$x = \frac{\pi}{4} + k\pi; \quad x = \frac{5}{6}\pi + k\pi$$

$$319 \operatorname{tg}\left(\frac{3}{2}\pi - x\right) = 1 - \operatorname{ctg}\left(\frac{\pi}{4} + x\right)$$

$$x = \frac{\pi}{4} + k\pi; \quad x = \operatorname{arcctg}(-2)$$

$$320 \frac{\sin 2x}{\sin x} + \cos\left(x - \frac{4}{3}\pi\right) = \sqrt{3} \sin x$$

$$x = \frac{\pi}{6} + k\pi$$

$$321 \sin\left(\frac{\pi}{6} + x\right) \cos\left(\frac{\pi}{3} + x\right) - \sin 2x = \frac{3}{2} \cos^2 x$$

$$x = -\frac{\pi}{4} + k\pi; \quad x = \operatorname{arctg}\left(-\frac{5}{3}\right)$$

$$322 3 \sin\left(\frac{\pi}{3} - 2x\right) + 2 \sin x \cos x + \cos\left(\frac{5}{6}\pi + 2x\right) = 1$$

$$x = \frac{\pi}{12} + k\pi; \quad x = -\frac{\pi}{4} + k\pi$$

**323**  $\frac{1}{2(1+\operatorname{ctg}^2 x)} - \cos^2\left(\frac{7}{4}\pi + x\right) = \cos\frac{2}{3}\pi$

**324**  $\operatorname{sen}x \cos(3\pi - x) + \cos\left(\frac{\pi}{4} + x\right) \operatorname{sen}\left(\frac{\pi}{4} - x\right) = 0$

**325**  $\operatorname{tg}2x + \operatorname{tg}\left(\frac{\pi}{4} - x\right) = 2$

**326**  $\operatorname{ctg}\left(x - \frac{\pi}{3}\right) + 2\operatorname{sen}\left(x + \frac{\pi}{6}\right) = 0$

**327**  $\cos 2x + 2\operatorname{sen}^2\left(x + \frac{3}{4}\pi\right) = 2$

**328**  $\cos^2\left(\frac{11}{6}\pi - x\right) + \frac{\sqrt{3}}{4}\operatorname{sen}2x = \cos^2 x$

**329**  $\frac{\cos 2x}{\sqrt{2} \cos\left(\frac{\pi}{4} - x\right)} + \operatorname{sen}\left(\frac{\pi}{2} - x\right) = 1$

**330**  $1 - 2\operatorname{sen}^2 x + \sqrt{3} \operatorname{sen}2x = (1 + \operatorname{ctg}^2 x) \cos^2\left(\frac{3}{2}\pi - x\right)$

**331**  $4 \operatorname{sen}^3 x - 4 \operatorname{sen}x \operatorname{sen}2x - 3 \operatorname{sen}x + 6 \cos x = 0$

**332**  $\cos 3x - \cos 5x = \operatorname{sen}6x + \operatorname{sen}2x$

**333**  $\frac{\sqrt{3} \cos x}{\operatorname{sen}\left(\frac{\pi}{3} - x\right)} + \frac{\operatorname{sen}x}{\cos\left(\frac{\pi}{6} + x\right)} = 1$

**334**  $2 - \cos 2x - 2 \operatorname{sen}^2 2x = 0$

**335**  $\frac{1}{\cos 2x} = 1 + \frac{\cos\left(\frac{\pi}{4} + x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$

## Sistemi goniometrici

Risolvere i seguenti sistemi.

$$\begin{cases} x = \frac{y}{2} \\ \sin^2 x + \cos^2 y = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{4} + k\frac{\pi}{2} \\ y = \frac{\pi}{2} + k\pi \end{cases}; \quad \begin{cases} x = \pm\frac{\pi}{6} + k\pi \\ y = \pm\frac{\pi}{3} + 2k\pi \end{cases}$$

$$\begin{cases} x + 2y = \frac{\pi}{3} \\ \sin \frac{x}{2} + \cos y = \sqrt{3} \end{cases}$$

$$\begin{cases} x = \frac{2}{3}\pi - 4k\pi \\ y = -\frac{\pi}{6} + 2k\pi \end{cases}$$

$$\begin{cases} x - y = \frac{\pi}{4} \\ \sin x + \sqrt{2} \cos y = 1 \end{cases}$$

$$\begin{cases} x = 2k\pi \\ y = -\frac{\pi}{4} + 2k\pi \end{cases}; \quad \begin{cases} x = \alpha + 2k\pi \\ y = \alpha - \frac{\pi}{4} + 2k\pi \end{cases} ; \quad \begin{cases} \alpha = \arcsen \frac{4}{5}; \quad \frac{\pi}{2} < \alpha < \pi \end{cases}$$

$$\begin{cases} x + y = \frac{3}{2}\pi \\ \sin x + \sin y = 1 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + 2k\pi \\ y = \pi - 2k\pi \end{cases}; \quad \begin{cases} x = \pi - 2k\pi \\ y = \frac{\pi}{2} + 2k\pi \end{cases}$$

$$\begin{cases} x - y = \frac{\pi}{3} \\ \sin x - \frac{\sqrt{3}}{2} \cos y = \frac{1}{4} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + 2k\pi \\ y = \frac{\pi}{6} + 2k\pi \end{cases}; \quad \begin{cases} x = \frac{7}{6}\pi + 2k\pi \\ y = \frac{5}{6}\pi + 2k\pi \end{cases}$$

$$\begin{cases} x - y = \frac{\pi}{3} \\ \sin x + \cos y = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = 2k\pi \\ y = -\frac{\pi}{3} + 2k\pi \end{cases}; \quad \begin{cases} x = \frac{5}{6}\pi + 2k\pi \\ y = \frac{\pi}{2} + 2k\pi \end{cases}$$

**342** 
$$\begin{cases} x + y = \pi \\ \sin 2x + \cos y = 0 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + k\pi \\ y = \frac{\pi}{2} - k\pi \end{cases}; \quad \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ y = \frac{5}{6}\pi - 2k\pi \end{cases}; \quad \begin{cases} x = \frac{5}{6}\pi + 2k\pi \\ y = \frac{\pi}{6} - 2k\pi \end{cases}$$

**343** 
$$\begin{cases} 2x + y = \frac{\pi}{4} \\ \cos x + \sin y = -\frac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + k\pi \\ y = -\frac{3}{4}\pi - 2k\pi \end{cases}; \quad \begin{cases} x = \frac{5}{12}\pi + 2k\pi \\ y = -\frac{7}{12}\pi - 4k\pi \end{cases}; \quad \begin{cases} x = \frac{13}{12}\pi + 2k\pi \\ y = -\frac{23}{12}\pi - 4k\pi \end{cases}$$

**344** 
$$\begin{cases} x - y = \frac{\pi}{6} \\ \sin x \cos y = \frac{1}{2} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + k\pi \\ y = \frac{\pi}{3} + k\pi \end{cases}; \quad \begin{cases} x = \frac{\pi}{6} + k\pi \\ y = k\pi \end{cases}$$

**345** 
$$\begin{cases} x - 2y = -\frac{3}{4}\pi \\ \cos x - \cos y = 0 \end{cases}$$

$$\begin{cases} x = \frac{3}{4}\pi + 4k\pi \\ y = \frac{3}{4}\pi + 2k\pi \end{cases}; \quad \begin{cases} x = -\frac{\pi}{4} + \frac{4}{3}k\pi \\ y = \frac{\pi}{4} + \frac{2}{3}k\pi \end{cases}$$

**346** 
$$\begin{cases} 2x + y = \pi \\ \operatorname{tg} 2x - \operatorname{tg} y = 2 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{8} + k\frac{\pi}{2} \\ y = \frac{3}{4}\pi - k\pi \end{cases}$$

**347** 
$$\begin{cases} x - y = \frac{\pi}{2} \\ 3\sin x - \cos y = \sqrt{2} \end{cases}$$

$$\begin{cases} x = \frac{3}{4}\pi + 2k\pi \\ y = \frac{\pi}{4} + 2k\pi \end{cases}; \quad \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ y = -\frac{\pi}{4} + 2k\pi \end{cases}$$

**348** 
$$\begin{cases} \sin x - \cos y = 1 \\ \sin^2 x - \cos^2 y = 0 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{6} + 2k\pi; x = \frac{5}{6}\pi + 2k\pi \\ y = \frac{2}{3}\pi + 2h\pi; y = \frac{4}{3}\pi + 2h\pi \end{cases}$$

**349** 
$$\begin{cases} \sin x + \cos y = 1 \\ \sin^2 x + 3\cos^2 y - 1 = 0 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{2} + 2k\pi \\ y = \frac{\pi}{2} + h\pi \end{cases}; \quad \begin{cases} x = \frac{\pi}{6} + 2k\pi; x = \frac{5}{6}\pi + 2k\pi \\ y = \pm\frac{\pi}{3} + 2h\pi \end{cases}$$

**350** 
$$\begin{cases} \log \sin x - \log \cos y = 0 \\ \sin x \cos y = \frac{3}{4} \end{cases}$$

$$\begin{cases} x = \frac{\pi}{3} + 2k\pi; x = \frac{2}{3}\pi + 2k\pi \\ y = \pm\frac{\pi}{6} + 2h\pi \end{cases}$$

**351** 
$$\begin{cases} 2^{\sin x} - 2^{-\cos y} = 0 \\ \sin^2 x + \cos^2 y = 1 \end{cases}$$

$$\begin{cases} x = \frac{\pi}{4} + 2k\pi; x = \frac{3}{4}\pi + 2k\pi \\ y = \pm\frac{3}{4}\pi + 2h\pi \end{cases}; \quad \begin{cases} x = -\frac{\pi}{4} + 2k\pi; x = -\frac{3}{4}\pi + 2k\pi \\ y = \pm\frac{\pi}{4} + 2h\pi \end{cases}$$